Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 1 (Group)

香港数学竞赛 **(2005 – 2006)**

决赛项目 **1 (**团体**)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 k 为实数。若 $x^2+2kx-3k^2$ 能被 $x-1$ 整除，求 k 最大可能的值。

Given that k is a real number. If $x^2+2kx-3k^2$ can be divisible by $x-1$, find the greatest value of k.

2. 已知 $x=x_0$ 及 $y=y_0$ 满足方程组 $\begin{cases}\frac{x}{3}+\frac{y}{5}=1\\ \frac{x}{5}+\frac{y}{3}=1\end{cases}$。若 $B=\frac{1}{x_0}+\frac{1}{y_0}$，求 B 的值。

Given that $x=x_0$ and $y=y_0$ satisfy the system of equations $\begin{cases}\frac{x}{3}+\frac{y}{5}=1\\ \frac{x}{5}+\frac{y}{3}=1\end{cases}$. If $B=\frac{1}{x_0}+\frac{1}{y_0}$, find the value of B.

3. 已知 $x=2+\sqrt{3}$ 是方程 $x^2-(\tan\alpha+\cot\alpha)x+1=0$ 的一个根。若 $C=\sin\alpha\times\cos\alpha$，求 C 的值。

Given that $x=2+\sqrt{3}$ is a root of the equation $x^2-(\tan\alpha+\cot\alpha)x+1=0$. If $C=\sin\alpha\times\cos\alpha$, find the value of C.

4. 设 a 为整数。若不等式 $|x+1|<a-1.5$ 没有整数解，求 a 最大可能的值。

Let a be an integer . If the inequality $|x+1|<a-1.5$ has no integral solution , find the greatest value of a .

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 2 (Group)

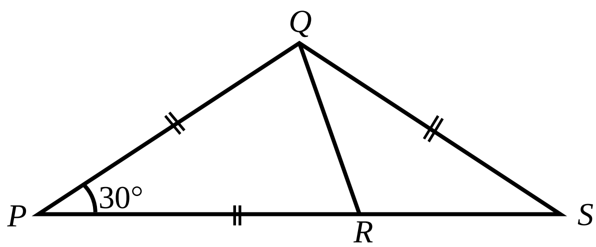
香港数学竞赛 (2005 – 2006)

决赛项目 2 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如图一，PRS 是一值线， $PQ = PR = QS$ 及 $\angle QPR = 30^\circ$。若 $\angle RQS = w^\circ$，求 w 的值。

In Figure 1 , PRS is a straight line , $PQ = PR = QS$ and $\angle QPR = 30^\circ$. If $\angle RQS = w^\circ$, find the value of w .



2. 设 $f(x) = px^7 + qx^3 + rx - 5$，其中 p 、 q 及 r 是实数。若 $f(-6) = 3$ 及 $z = f(6)$，求 z 的值。

Let $f(x) = px^7 + qx^3 + rx - 5$, where p , q and r are real numbers . If $f(-6) = 3$ and $z = f(6)$, find the value of z .

3. 若 $n \neq 0$ 及 $s = \left(\dfrac{20}{2^{2n+4} + 2^{2n+2}}\right)^{\frac{1}{n}}$，求 s 的值。

If $n \neq 0$ and $s = \left(\dfrac{20}{2^{2n+4} + 2^{2n+2}}\right)^{\frac{1}{n}}$, find the value of s .

4. 已知 x 和 y 是正整数及 $x + y + xy = 54$。若 $t = x + y$，求 t 的值。

Given that x and y are positive integers and $x + y + xy = 54$. If $t = x + y$, find the value of t .

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 3 (Group)
香港数学竞赛 (2005 – 2006)
决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $r=2006\times\frac{\sqrt{8}-\sqrt{2}}{\sqrt{2}}$，求 r 的值。

Given that $r=2006\times\frac{\sqrt{8}-\sqrt{2}}{\sqrt{2}}$, find the value of r.

2. 已知 $6^{x+y}=36$ 及 $6^{x+5y}=216$，求 x 的值。

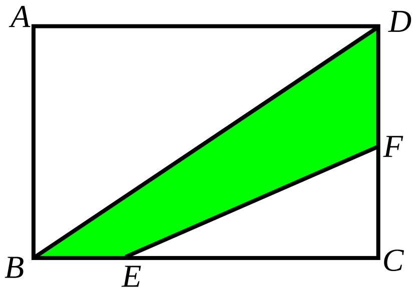
Given that $6^{x+y}=36$ and $6^{x+5y}=216$, find the value of x.

3. 已知 $\tan x+\tan y+1=\cot x+\cot y=6$。若 $z=\tan(x+y)$，求 z 的值。

Given that $\tan x+\tan y+1=\cot x+\cot y=6$. If $z=\tan(x+y)$, find the value of z.

4. 如图一，$ABCD$ 是一长方形，F 是 CD 的中点及 $BE:EC=1:3$。若长方形 $ABCD$ 的面积是 12 cm^2 及阴影部分 $BEFD$ 的面积是 $R\text{ cm}^2$，求 R 的值。

In Figure 1, $ABCD$ is a rectangle, F is the midpoint of CD and $BE:EC=1:3$. If the area of the rectangle $ABCD$ is 12 cm^2 and the area of $BEFD$ is $R\text{ cm}^2$, find the value of R.



图一
Figure 1

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 4 (Group)

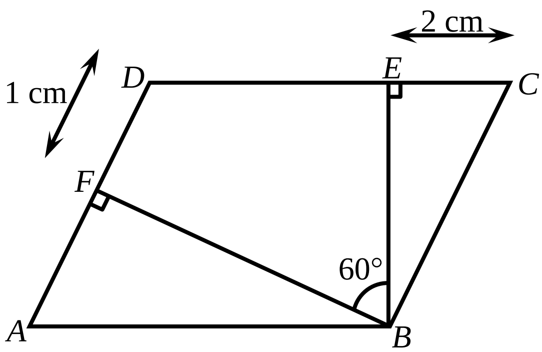
香港数学竞赛 (2005 – 2006)

决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如图一，平行四边形 $ABCD$ 中，$BE \perp CD$，$BF \perp AD$，$CE = 2$ cm，DF = 1 cm 及 $\angle EBF = 60^\circ$。若平行四边形 $ABCD$ 的面积是 R cm^2，求 R 的值。

In Figure 1 , $ABCD$ is a parallelogram , $BE \perp CD$, $BF \perp AD$, $CE = 2$cm, DF = 1 cm and $\angle EBF = 60^\circ$. If the area of the parallelogram $ABCD$ is R cm^2 , find the value of R .



图一

Figure 1

2. 已知 a 和 b 是正实数且 $a + b = 2$。若 $S = \left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2$，求 S 的最小值。

Given that a and b are positive real numbers and $a + b = 2$. If $S = \left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2$, find the minimum value of S .

3. 设 $2^x = 7^y = 196$。若 $T = \frac{1}{x} + \frac{1}{y}$，求 T 的值。

Let $2^x = 7^y = 196$. If $T = \frac{1}{x} + \frac{1}{y}$, find the value of T .

4. 若 $W = 2006^2 - 2005^2 + 2004^2 - 2003^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$，求 W 的值。

If $W = 2006^2 - 2005^2 + 2004^2 - 2003^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$, find the value of W .